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# Building Monotonicity-Preserving Fuzzy Inference Models with Optimization-Based Similarity Reasoning and a Monotonicity Index

*[1]Kai Meng Tay, [2]Chee Peng Lim, [1]Tze Ling Jee

[1]Faculty of Engineering, Universiti Malaysia Sarawak,
Sarawak, Malaysia
*kmtay@feng.unimas.my

[2]School of Computer Sciences
University of Science Malaysia
Malaysia

***Abstract*—In this paper, a novel approach to building a Fuzzy Inference System (FIS) that preserves the monotonicity property is proposed. A new fuzzy re-labeling technique to re-label the consequents of fuzzy rules in the database (before the Similarity Reasoning process) and a monotonicity index for use in FIS modeling are introduced. The proposed approach is able to overcome several restrictions in our previous work that uses mathematical conditions in building monotonicity-preserving FIS models. Here, we show that the proposed approach is applicable to different FIS models, which include the zero-order Sugeno FIS and Mamdani models. Besides, the proposed approach can be extended to undertake problems related to the local monotonicity property of FIS models. A number of examples to demonstrate the usefulness of the proposed approach are presented. The results indicate the usefulness of the proposed approach in constructing monotonicity-preserving FIS models.**

***Keywords-Fuzzy inference system; monotonicity property; local monotonicity; monotonicity index; sufficient conditions; similarity reasoning.***

## I. INTRODUCTION

Consider a fuzzy inference system (FIS) model, $y = f(\bar{x};\ \theta)$, where ($\bar{x} = x_1, x_2, \ldots, x_n$ ) is a vector and is simplified as $x_k$, $k = 1,2,3,\ldots,n$, and θ is a parameter vector that describes the mathematical model. For an FIS model, θ parameterizes the fuzzy membership functions (MFs), fuzzy rule base, consequents, and etc. On one hand, for an FIS model that fulfils the *monotonicity* condition between its output, *y*, and the *i*-th input (*i* is a subset of *k*), $x_i$, within the universe of discourse, $y$ monotonically increases or decreases as $x_i$ increases, i.e., $f(x_{k\neq i}, x_i^1,) \leq f(x_{k\neq i}, x_i^2)$ or $f(x_{k\neq i}, x_i^1,) \geq f(x_{k\neq i}, x_i^2)$ respectively, for $x_i^1 < x_i^2$ within the universe of discourse. On the other hand, for an FIS model that fulfils the *local monotonicity* condition between $y$ and $x_i$ within the upper and lower bounds of $\overline{x_i}$ and $\underline{x_i}$, respectively (where the limit between the upper and lower bound is a subset of the universe discourse), $y$ monotonically increases or decreases as $x_i$, increases, i.e., $f(x_{k\neq i}, x_i^1,) \leq f(x_{k\neq i}, x_i^2)$ or $f(x_{k\neq i}, x_i^1,) \geq f(x_{k\neq i}, x_i^2)$ respectively, for $x_i^1 < x_i^2$ within the limit. As an example, a quadratic function, $y = x^2$ observes the local monotonicity condition.

The importance of the monotonicity property in FIS modeling has been highlighted in a number of recent publications [1-7]. However, the study of the local monotonicity property in FIS modeling is new. The idea of local monotonicity regression was introduced in the literature [8-10]. Its importance in filter and signal processing has also been highlighted [8-10]. In this paper, the problems of preserving the monotonicity and local monotonicity properties are addressed as the *monotone fuzzy modeling problem*. The monotone fuzzy modeling problem includes constructing an FIS model (type 1 or type 2) either manually or with machine learning techniques, tuning/ optimizing an FIS model [11], and developing a new mathematical framework to facilitate/complement the FIS modeling process, e.g., Similarity Reasoning (SR) [12] and re-labeling [13]. The monotonicity or local monotonicity relationships among the input(s) and the output of a system constitute useful qualitative information in FIS modeling.

A number of investigations of the *monotone fuzzy modeling problem* exist. These include the development of mathematical conditions as a set of governing equations for an FIS model to observe the monotonicity property [1, 2, 6]. Application of these developed mathematical conditions to real-world problems has been reported [3-5, 7]. These mathematical conditions have also been extended to some advanced FIS modeling techniques [11-12]. From the literature, mathematical conditions or guidelines for the Sugeno FIS model (i.e., the *sufficient conditions* (as explained in Section II(b) )) [1], Mamdani FIS model [6] and SIRM FIS model [2], have been developed. In [11], a monotonicity index has been proposed. It can be used to evaluate, approximately, the monotonicity property of an FIS model. It also provides useful information pertaining to the FIS model, either it fulfills the monotonicity property, or to what extend it fulfills the monotonicity property.

In this study, SR is viewed as a computing paradigm, as shown in Fig. 1. SR has been proposed as a solution to undertake the incomplete rule base problems.

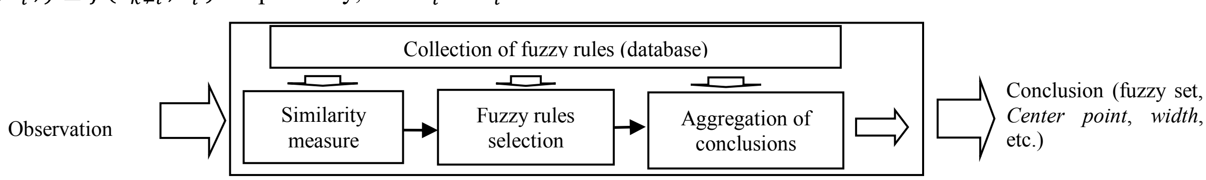


Figure 1 A similarity reasoning paradigm